

SEISMIC DESIGN OF RETAINING WALLS
WITH REFERENCE TO THE
MASSACHUSETTS STATE BUILDING CODE

by

Cetin Soydemir, Ph.D., Senior Engineer
Haley & Aldrich, Inc., Massachusetts, U.S.A.

ABSTRACT

An overview of the current methods for the seismic design of retaining walls was conducted in reference to the criterion specified in the Massachusetts State Building Code (1979). The particular nature of the soil (backfill) vs. structure (wall and connecting structural members) interaction has been underlined as the governing element in predicting earthquake induced dynamic loads on retaining walls. Available and readily usable elastic and rigid-plastic models may be appropriate to predict dynamic thrust associated with "small" and "large" wall deformations (movements), respectively. For the seismic design of gravity walls, recently proposed criteria based on a limiting wall displacement should be adopted.

INTRODUCTION

Massachusetts State Building Code (1), the Code, Section 716.0 (EARTHQUAKE LOAD), Subsection 716.6.10 (RETAINING WALLS) specifies that "Retaining walls shall be designed to resist at least the superimposed effects of the total static lateral soil pressure plus an earthquake force of $0.045 \gamma_t H^2$ (horizontal backfill surface)...the earthquake force from the backfill shall be distributed as an inverse triangle over the height of the wall...for non-liquefaction condition" (γ_t = unit weight of backfill, H = height of wall).

In Subsection 716.7 (DYNAMIC ANALYSIS), the Code (1) prescribes a design earthquake representative of the regional seismicity as "... an earthquake with a peak acceleration of 0.12 g."

MODELS FOR SEISMIC DESIGN OF RETAINING WALLS

Common types of retaining walls are depicted in Figure 1. It is expected that the different wall-backfill systems are likely to behave differently when subjected to ground shaking. The current models for

seismic design of retaining walls may be grouped in two categories: a) models based on a force criterion, b) models based on a displacement criterion.

Models Based on Force Criterion

The most common force-criterion model used in design practice was introduced over half a century ago by Mononobe (2) and Okabe (3). The so-called Mononobe-Okabe (M-O) model is an ingenious but somewhat arbitrary extension of the Coulomb (1776) model developed for the static loading condition. The assumptions involved in the M-O model were stated and critically reviewed by Seed and Whitman (4), Wood (5), Whitman (6), Nadim and Whitman (7), etc. M-O model for "active" condition formulates the total lateral force, P_{AE} (i.e., initial static load, P_A , plus subsequent incremental dynamic thrust due to ground shaking, P_D) as:

$$P_{AE} = P_A + P_D = 1/2 \gamma H^2 K_{AE} \quad (1)$$

where

$$K_{AE} = \frac{\cos^2 (\phi - \theta - \beta)}{\cos \theta \cos^2 \beta \cos (\delta + \beta + \theta)} \left[1 + \left(\frac{\sin (\phi + \delta) \sin (\phi - \theta - i)}{\cos (\delta + \beta + \theta) \cos (i - \beta)} \right) 1/2 \right]^2 \quad (2)$$

$$\text{with} \quad \tan \theta = k_h / (1 - k_v) \quad (3)$$

k_h and k_v are the seismic design acceleration coefficients in the horizontal and vertical directions, respectively. Other parameters stated in Eqs. 1 and 2 are defined in Figure 2. It is apparent that the dynamic force component, P_D , specified in the Code (1) is based on a simplified form of the M-O model, proposed by Seed and Whitman (4):

$$P_D = 1/2 \gamma_t H^2 \Delta K_{AE} \approx 1/2 \gamma_t H^2 (3/4 k_h) = 3/8 \gamma_t H^2 k_h \quad (4)$$

Substituting $k_h = 0.12$ for the design earthquake in Massachusetts:

$$P_D = 0.045 \gamma_t H^2 \quad (5)$$

is obtained.

In its original presentation of the M-O formulation (2,3), the position of the dynamic thrust was not explicitly stated. However, it is likely that a triangular dynamic pressure distribution was presumed. Based on the results of theoretical and experimental work reported by several researchers (1930-1970), Seed and Whitman (4) recommended that the line of action of the dynamic force, P_D , be taken at $2/3 H$ above the base of the wall. This recommendation was also adopted in the Code (1).

Past design practice has disclosed that direct substitution of horizontal peak ground acceleration ($a_{\max} = k_h g$) into Eq. 3 usually leads to relatively massive, thus costly, wall sizes. Therefore, it has become customary to use a k_h value obtained through an arbitrary reduction in a_{\max} . This is a fundamental weakness of the M-O model. M-O model is associated with the limiting equilibrium state (i.e., active or passive) of the soil backfill retained by the wall. In parallel, it is a rigid-plastic model.

At the Second World Conference on Earthquake Engineering (1960) Matuo and Ohara (8) proposed one of the few design methods available, which incorporates saturated (non-liquefied) as well as unsaturated backfills. The method utilized elastic wave propagation theory and laboratory model test results, and was developed primarily for the seismic design of quay walls used extensively in Japan. It was described in detail by Seed and Whitman (4).

At the Fifth World Conference on Earthquake Engineering (1973), Scott (9) presented an analytical method to predict the dynamic pressure distribution behind a rigid-nonyielding wall. In the method, the backfill was modelled as a one-dimensional shear beam supported on a rigid base. The shear beam is connected to a rigid wall by elastic springs which are representative of the soil-wall interaction.

Wood's (1973) doctoral thesis (5) is an extensive study of the seismic behavior of rigid retaining walls on firm base, utilizing theory of elasticity. Wood (5) presented both pseudo-static (incorporating a static horizontal body force field), and dynamic analytical and finite element solutions. Wood's solutions (Fig. 3) illustrate the significant difference in earthquake induced lateral earth pressures for the cases of elastic and rigid-plastic (M-O) models. Wood (5) demonstrated how a particular soil-structure interaction, associated with a "small" or "large" strain-field in the backfill, contributes to the dynamic pressure intensity and distribution. In Figure 3, M-O pressure distribution is assumed (by the author) linearly increasing with depth for the purpose of comparison. Figure 3b was developed for a horizontal acceleration of 1.0 g, for convenience. By direct proportioning, pressure distributions may be obtained for different levels of acceleration. Figure 3b illustrates the significant effect of yielding of the wall (in the elastic range) on the induced dynamic earth pressure (i.e., magnitude and distribution wise).

Utilizing dynamic finite element analyses which incorporated strain compatible modulus and damping, Idriss and Seed (10, 11) investigated the case of a single massive structure embedded at a relatively great depth (80 ft.) and proposed a simple procedure for calculating the dynamic pressure behind a basement-type wall. They suggested that the dynamic thrust can be reasonably estimated by the simplified Seed-Whitman (4) formulation (Eq. 4):

$$P_D = 3/8 \gamma_t H^2 k_h$$

where ($k_h g$) is the ground surface acceleration. The dynamic pressure distribution was found to be nearly rectangular (i.e., the dynamic

thrust acting at $H/2$ above the base of the wall).

The so-called "apparent seismic coefficient method" is adopted in current design practice in Japan (12) to estimate the total lateral thrust (P_{AE}) exerted by a saturated backfill. In using this approach, the designer first determines the apparent seismic coefficient k_h' :

$$k_h' = \gamma_t k_h / \gamma_b \quad (6)$$

where γ_t and γ_b are saturated and buoyant unit weight of the backfill, respectively. Then, M-O coefficient K_{AE} (Eq. 2) is computed with k_h taken as k_h' . Finally, P_{AE} (Eq. 1) is calculated taking $\gamma = \gamma_b$, and static hydrostatic pressure is added. The author has established that dynamic thrust (P_D) for saturated backfills estimated by the Matuo-Ohara (8) formulation and "apparent seismic coefficient method" (12) are in close agreement for $k_h \leq 0.2$. However, for $k_h > 0.2$, dynamic thrust computed by the "apparent seismic coefficient method" increases at a somewhat questionable rate.

Models Based on Displacement Criterion

In 1979, Richards and Elms (13) proposed a procedure for earthquake design of gravity retaining walls based on the observation that a gravity wall does not "fail" when the base ground acceleration reaches a critical level (i.e., factor of safety against sliding drops to 1.0), but rather it experiences a finite permanent displacement relative to the base ground. The sliding-block analogy used in the Richards-Elms (R-E) model was originally introduced by Newmark (14). In using the R-E approach, designer first determines an allowable permanent displacement (d) for the wall. Then he uses (d) to compute a design acceleration (N_g) through:

$$d \text{ (inch)} = 0.087 (V^2/Ag) (N/A)^{-4} \quad (7)$$

where (N_g) is the cut-off acceleration above which the wall is initiated to move relative to the base ground. A (inch per sec. sq.) and V (inch per sec.) are the maximum acceleration and maximum velocity representative of the regional seismicity. For convenience, Richards and Elms inverted Eq. 7:

$$N = A_d [0.2 A_v^2 / A_a d]^{\frac{1}{4}} \quad (8)$$

where A_a and A_v are dimensionless parameters defined as effective peak acceleration coefficient and velocity related acceleration coefficient, respectively (15). In obtaining Eq. 8, Richards and Elms also used the empirical correlation V (inch per sec.) = 30 A_v , as recommended by the Applied Technology Council (15). Values of A_a and A_v are given in Ref. 15 for regions of the United States. Along similar lines, Whitman (6) used average limiting values of $V/A = 1250$ mm per sec. (soft base ground) and $V/A = 750$ mm per sec. (very firm base) in evaluating N by Eq. 7.

The final step of the R-E approach is to calculate P_{AE} , utilizing M-O formulation (Eq. 1, 2) with $N = k_h$. Incorporating the effect of the wall inertia as well, Richards and Elms (13) derived an expression for the required wall weight (W_w) for the limiting equilibrium condition

(i.e., factor of safety against sliding being 1.0):

$$W_w = \frac{1/2 \gamma H^2 [\cos(\delta + \beta) - \sin(\delta + \beta) \tan \phi_b]}{\tan \phi_b - \tan \theta} K_{AE} \quad (9)$$

where (ϕ_b) is the angle of friction between the wall and the base ground. It is recommended that a factor of safety of 1.2 to 1.3 be used for the design weight of the wall (16).

Since 1979, design models based on limiting displacement criterion have been further investigated both at the University of Canterbury, New Zealand and Massachusetts Institute of Technology, Cambridge, Massachusetts (M.I.T.). Although an effective design tool, R-E model (13) incorporates certain simplifying assumptions which contribute to the level of approximation in obtaining (N_g). For example, the model assumes a constant value of wall acceleration (N_g) while slippage occurs. It does not account for vertical acceleration and only sliding mode of displacement is considered, neglecting displacement associated with tilting. In addition, the backfill is assumed to behave in a rigid-plastic manner such that potential amplification in the backfill is not taken into account.

Zarrabi (17) investigated the seismic behavior of gravity walls by developing a rigid-plastic model which treats the soil wedge as a second sliding block, separate from the wall. At every time increment, the equilibrium and compatibility requirements along the wall and a selected failure plane are satisfied by recalculating the inclination of the failure plane. This way, Zarrabi (17), was able to account for the time-varying dynamic earth pressure. Zarrabi's (17) analyses indicated that the effect of excluding vertical ground acceleration in estimating permanent wall displacement is negligible at "low" N/A values; however, for "high" N/A values, the error may be 10 to 30 percent on the unsafe side. Nadim (18) extended Zarrabi's model by incorporating tilting mode of displacement (i.e., rotation). He concluded that tilting motion of the wall is not important as long as the wall starts to slide before tilting is initiated (i.e., N/A for sliding is smaller than N/A for tilting).

Wong and Whitman (19) using both Newmark (14) and Zarrabi (17) sliding block models investigated the effects of wall orientation (i.e., with respect to ground shaking), vertical acceleration in the backfill and ground motion characteristics. They concluded that the orientation of the wall has a significant role in the prediction of permanent wall displacement as well as being an important parameter in assessing the uncertainty in the estimated displacement value. Wong and Whitman (19) also pointed out that due to different characteristics of the chosen design strong motion accelerograms (with respect to three orthogonal directions and time), the mean normalized wall displacement will be different for each earthquake, and thus a collection of random data will be obtained for the same wall at a particular orientation. Wong and Whitman (19) incorporating orientation of the wall, ground shaking characteristics of a number of selected strong motion accelerograms and

vertical acceleration proposed a "prediction rule" to predict permanent wall displacement, d :

$$d = [(37v^2)/(Ag)] e^{-9.4(N/A)} R_v R_z \quad (10)$$

where R_v is a vertical acceleration factor, and R_z is a factor to correlate Newmark's model to Zarrabi's model (R_v and R_z are both functions of N/A). Wong and Whitman (19) estimated that the confidence level of the proposed "prediction rule" is about 90 to 95 percent, thus a further factor of safety need not be supplied to the weight of the wall estimated by Eq. 9.

Antia and Whitman (20) investigated the effects of wall inclination, wall friction and inclination of sloping backfill on the predicted wall displacement value by the M.I.T. model (Eq. 10) and provided directly applicable correction factors for these parameters in design.

Previously discussed models because of their rigid-plastic character could not take into account amplification of ground motion in the backfill. Nadim and Whitman (7) developed an "elastic" finite element model with special frictional elements incorporated along the wall and pre-selected failure surfaces, which allow the wall to tilt as well as to slide. With their model, Nadim and Whitman (7) studied the seismic behavior of rigid-nonyielding walls. Their prediction of the dynamic pressure was in agreement with Wood's (5) prediction, but the estimated dynamic thrust was about 30 percent higher than that predicted by the simple Idriss-Seed (10) model. The line of action of the dynamic thrust, however, was found the same to that suggested by Idriss and Seed (10) (i.e., $1/2 H$ above the base of the wall). Nadim and Whitman (7) reasoned that in predicting wall displacement, the amplification effect of the backfill is to be included in the seismic parameters (A) and (V) (Eq. 10 or Eq. 7). They proposed a simple procedure which evaluates the fundamental frequency of the backfill (f_1) (i.e., $f_1 = v_s/4H$, where v_s = shear wave velocity representative of the backfill), and compares it with the dominant frequency (f) of the expected base ground motion. It was recommended that for $f/f_1 \approx 0.25$ the amplification effect of the backfill may be neglected. For about $f/f_1 \approx 0.5$, A and V parameters are to be increased by 25 to 30 percent; and in the range of $0.7 \leq f/f_1 \leq 1.0$, A and V are to be increased by 50 percent.

SUMMARY - CONCLUSIONS

In seismic design of retaining walls, designer should have an intimate understanding of the particular soil-structure interaction involved (Figure 1). Where potential wall displacement is expected to be none to "very small", an elastic model i.e., Wood (5), Scott (9), Nadim and Whitman (7) should be preferably considered. This would yield a more representative dynamic pressure magnitude and distribution. On the other hand, if the wall displacement is expected to be "large" (e.g., greater than 0.005 to $0.01 H$), then a rigid-plastic analysis, such as the Mononobe-Okabe model, would be appropriate to predict the magnitude of the dynamic thrust. However, the line of action of dynamic thrust should be taken at $0.5 H$ to $0.6 H$ above the base of the wall. For the range of wall displacement between elastic and plastic behavior, a non-

linear elastic finite element analysis may be the rigorous approach, however, such an analysis is difficult to perform and costly. An approximate approach may be to utilize the elastic and rigid-plastic models to predict the two limiting conditions, and reach a conclusion, considering the over-all requirements of the particular project.

In seismic design of gravity retaining walls, it is well established that a limiting displacement criterion be adopted. Based on a predetermined allowable permanent wall displacement, a rational seismic coefficient can be estimated utilizing the Richards and Elms and or M.I.T. model. The estimated seismic coefficient may then be incorporated in Mononobe-Okabe model to predict the magnitude of the dynamic thrust. The line of action of the dynamic thrust should be taken at 0.5 H to 0.6 H above the base of the wall.

In the light of this study, it is the author's opinion that the requirements specified in the current Massachusetts State Building Code for the seismic design of retaining walls are on the average reasonable, especially for walls under 20 ft. in height.

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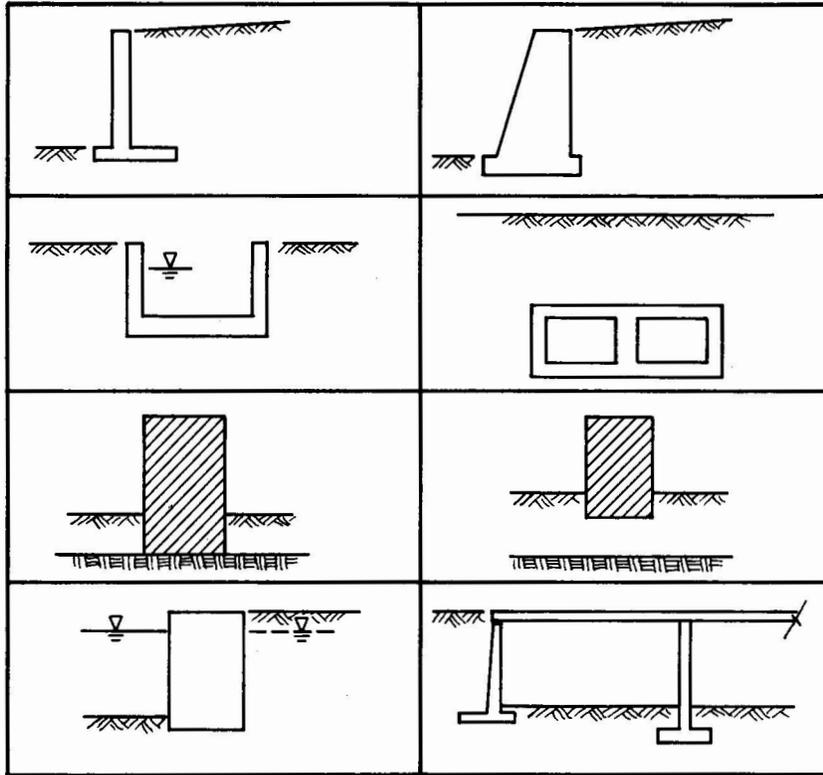


FIGURE 1. COMMON TYPES OF RETAINING WALLS

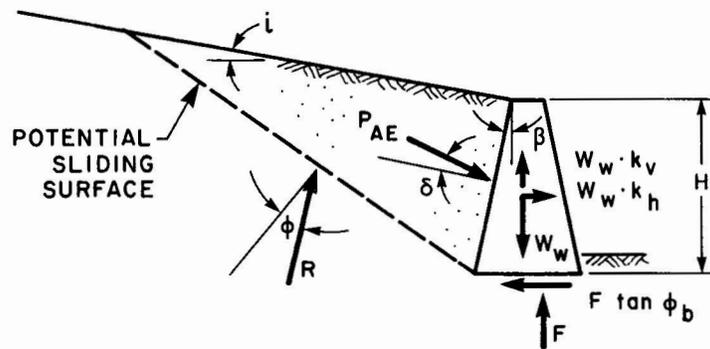


FIGURE 2. SEISMIC ACTIVE LIMITING EQUILIBRIUM CONDITION

— WOOD(5) ELASTIC MODEL:
 $\nu = 0.40, K_0 = 0.67, k_h = 0.33$

--- MON.-OKABE RIGID-PLASTIC MODEL:
 $k_h = 0.33$

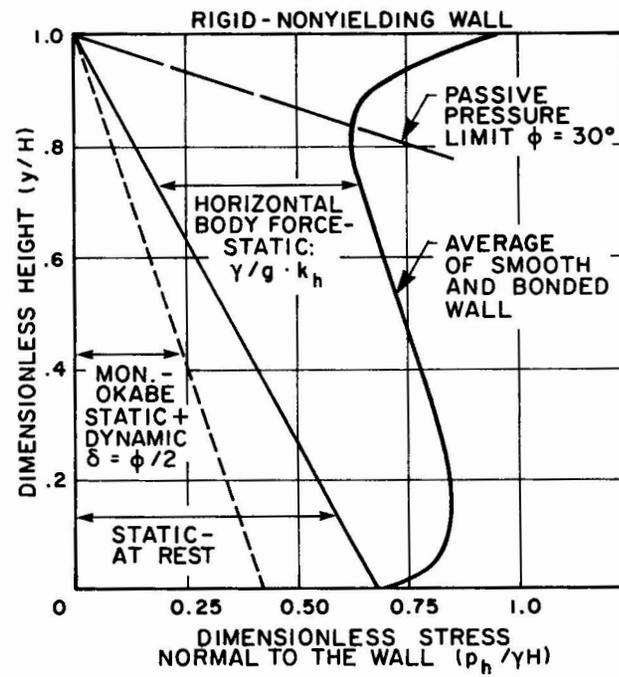


FIGURE 3a

— WOOD(5) ELASTIC MODEL:
 HORIZONTAL BODY FORCE ($\gamma/g \cdot k_h$)
 WITH $k_h = 1.0$

--- MON.-OKABE ($k_h = 1.0$)

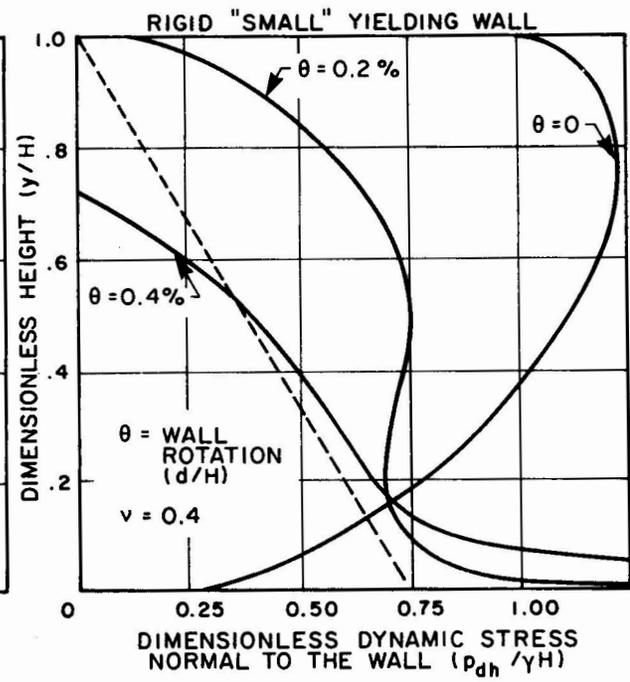


FIGURE 3b